Warm-up!

Find the inverse of the following functions:

A) $f(x) = 3\log_2(x+4)$

$x = 3\boxed{\log_2(y+4)}$

$2^{\left(\frac{x}{3}\right)} = 2^{\left(\log_2(y+4)\right)}$

$2^{\frac{x}{3}} = y+4$

$f^{-1}(x) = 2^{\frac{x}{3}} - 4$

$\sqrt[3]{2^x} - 4$

B) $g(x) = 2 - \ln\left(\frac{x}{5}\right)$

$x = 2 - \boxed{\ln\left(\frac{y}{5}\right)}$

$-2 + x = -\ln\left(\frac{y}{5}\right)$

$e^{(2-x)} = e^{\left(\ln\left(\frac{y}{5}\right)\right)}$

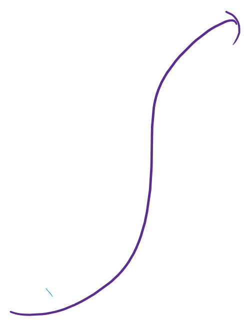
$e^{2-x} = \frac{y}{5}$

$5e^{2-x} = y$

$g^{-1}(x) = 5e^{2-x}$

$5e^{-x+2}$

$5\,\frac{e^2}{e^x}$



Log Functions Day 6:

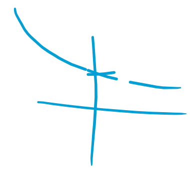
- (2.13) Inverse of Log Functions
- (2.13) Exponential Inequalities
- (2.13) Log Inequalities

Homework: Exponential and log inequalities

Let $f(x) = 2 - \log_3(x - 5)$

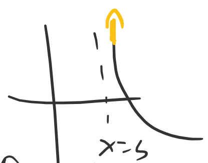
Find $f^{-1}(x)$

$$x = 2 - \log_3(y-5)$$
$$x - 2 = -\log_3(y-5)$$
$$-x + 2 = \log_3(y-5)$$
$$3^{-x+2} = y - 5$$
$$3^{-x+2} + 5 = y$$
$$f^{-1}(x) = 3^{-x+2} + 5$$



Find the vertical asymptote of $f(x)$. Describe this asymptote with a limit statement.

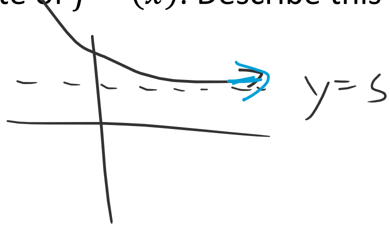
$x = 5$



$$\lim_{x \to 5^+} f(x) = \infty$$

Find the horizontal asymptote of $f^{-1}(x)$. Describe this asymptote with a limit statement.

$y = 5$



$$\lim_{x \to \infty} f^{-1}(x) = 5$$

Let $g(x) = 5\log(20 - x)$

Find $g^{-1}(x)$

Find the domain and range of g

Find the domain and range of g^{-1}

Find $g(10)$

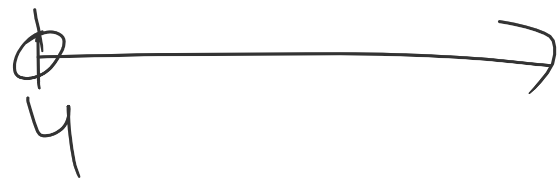
Find $g^{-1}(10)$

Strategy for All Inequality Problems

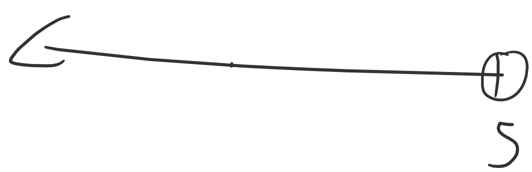
(Before you begin, draw a number line that reflects the domain of any function in the inequality. Cross off/erase parts of the number line that are not in the domain)

1. Solve the equality
2. Put solutions on number line
3. Test regions

$$\log_2(x-4) > 18$$



$$\frac{1}{2}\ln(5-x)+3<7$$



$$\frac{1}{8}\log_2(x+1) - 3\log_2(7-x) > 0$$

$x+1>0$

$x>-1$

$x<7$

Let $f(x) = \log_2(11 - x)$ and $g(x) = \log_2(x - 5) - 1$

Find all x-values for which $f(x) \geq g(x)$

(Before you begin, draw a number line that reflects the domain of any function in the inequality. Cross off/erase parts of the number line that are not in the domain)

1. Solve the equality
2. Put solutions on number line
3. Test regions

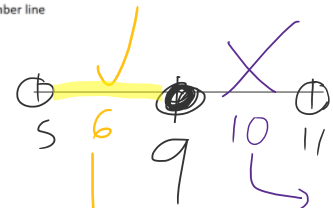
$\log_2(11-x) \geq \log_2(x-5) - 1$

$11 - x > 0$

$x < 11$

$x - 5 > 0$

$x > 5$



$\log_2(11-x) = \log_2(x-5) - 1$

$\log_2(11-x) - \log_2(x-5) = -1$

$2^{\left(\log_2\left(\frac{11-x}{x-5}\right)\right)} = 2^{(-1)}$

$\frac{11-x}{x-5} = \frac{1}{2}$

$22 - 2x = x - 5$

$27 = 3x$

$9 = x$

$\log_2(1) \geq \log_2(5) - 1$

$0 \geq 2.\text{something} - 1$

$\log_2 5 \geq \log_2 1 - 1$

$2.\text{something} \geq 0 - 1$

$(5, 9]$

Let $f(x) = \log$ and let

What are all values of x for which $f(x) \geq g(x)$?

(Before you begin, draw a number line that reflects the domain of any function in the inequality. Cross off/erase parts of the number line that are not in the domain)

1. Solve the equality
2. Put solutions on number line
3. Test regions

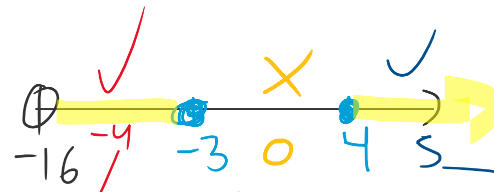
$$\log_2(x^2+4) \geq \log_2(x+16)$$

$$2^{\log_2(x^2+4)} = 2^{\log_2(x+16)}$$

$$x^2+4 = x+16$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$



$$\log_2 4 \geq \log_2 16$$

$$2 \geq 4$$

$$\log_2(20) \geq \log_2(12)$$

$$\log_2(29) \geq \log_2(21)$$

$$(-16, -3] \cup [4, \infty)$$

Let $f(x) = 8^{2x-1}$ and let $g(x) = 4^{x-5}$

Find the interval(s) over which $f(x) < g(x)$

$$8^{2x-1} < 4^{x-5}$$

$$8^{2x-1} = 4^{x-5}$$

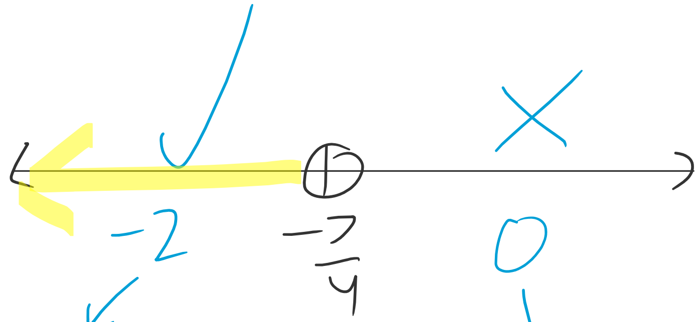
$$(2^3)^{2x-1} = (2^2)^{x-5}$$

$$2^{6x-3} = 2^{2x-10}$$

$$6x-3 = 2x-10$$

$$4x = -7$$

$$x = -7/4$$



$$8^{-5} < 4^{-7}$$

$$(2^3)^{-5} < (2^2)^{-7}$$

$$2^{-15} < 2^{-14}$$

$$\frac{1}{2^{15}} < \frac{1}{2^{14}}$$

$$8^{-1} < 4^{-5}$$

$$\frac{1}{8} < \frac{1}{4^5}$$

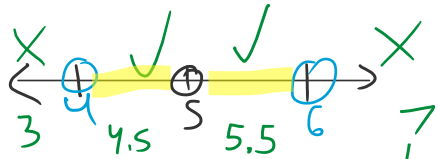
$$\frac{1}{8} < \frac{1}{1024}$$

Let $f(x) = \log_2(|x-5|)$

$(4,5) \cup (5,6)$

Find all values of x for which the output of $f(x)$ is negative

$\log_2(|x-5|) < 0$



$2^{\log_2(|x-5|)} = 2^{0}$

$\log_2(2) < 0$

$\log_2(.5) < 0$

$|x-5| = 1$

$\log_2(.5) < 0$

$x-5=1$ $\quad$ $x-5=-1$

$x=6$ $\quad$ $x=4$

$\log_2(2) < 0$

Let $f(x) = 3(2)^{1-x} + 6$

Find all x-values for which $f(x) \geq 15$

$$3(2)^{1-x} + 6 \geq 15$$

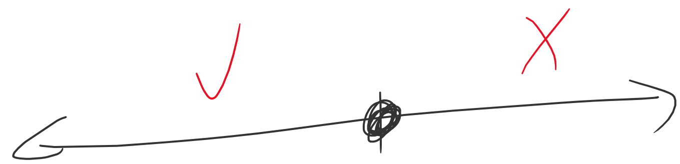
$$3(2)^{1-x} = 9$$

$$(2)^{1-x} \geq 3$$

$$\log_{\frac{1}{2}}\left(\left(\tfrac{1}{2}\right)^{x-1}\right) \geq \log_{\frac{1}{2}}(3)$$

$$x - 1 \geq \log_{\left(\frac{1}{2}\right)} 3$$

$$x \geq 1 + \log_{\frac{1}{2}} 3$$



-2 $\quad 1 + \log_{\frac{1}{2}}(3) \quad$ 10

$$3(2)^{3} + 6 \geq 15$$

$$30 \geq 15$$

$$3(2)^{-9} + 6 \geq 15$$